

# Two-Dimensional Analytical Model and Control of Linear Induction Motor

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## Research Article

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## ABSTRACT

Through electromagnetic forces, Linear Electric (Electromagnetic) Machines (LEMs) can directly convert electrical energy into linear mechanical kinetic energy (*vice versa*). Linear motion is especially common in industry. LEM was developed in his 19<sup>th</sup> century, but did not become widely used in industry until 1960 because it required power electronics (no mechanical transmission) for control. However, LEMs require power electronics for linear position, speed, and/or force control to achieve better performance than rotary electric motors with mechanical transmissions. After 1960, LEM continued to improve. The lift and drive forces of a two-phase Linear Induction Motor (LIM) are controlled by varying the phase angle between the two phases. In this article, we derive the magnetic flux density, secondary current density, and propulsion and levitation force densities in the air gap. The mean force equation is derived by summing the force density over a quadratic length using two-dimensional magnetic field analysis and is used to simulate the performance of a linear induction motor. We investigate the effect of changing the phase angle on the secondary current density, perpendicular magnetic flux density component, and tangential magnetic flux density component. Furthermore, these phase-shifting effects are extended to include his two components of force (levitation and propulsion). Calculations are performed using MATLAB programs and displayed graphically.

**Keywords:** Density; Magnetic flux; Magnets; Electric

## LIST OF ABBREVIATIONS

MAGLEV-Magnetically Levitated; LEM-linear electric (electromagnetic) machines; LIMs-linear induction motors; PMs-Permeant Magnets

INTRODUCTION

Depending on the method used for magnetic field analysis, the electrical stress on a magnetic circuit is usually expressed in the form of a Fourier series or Fourier integral. It is clear that determining the motor force requires knowledge of the excitation secondary current density and the air gap magnetic flux density components. The current density and magnetic flux density components are fully determined by solving the magnetic vector potential equations and applying boundary conditions. It must be expressed in a format suitable for analysis. The use of a Linear Induction Motor (LIM) is considered an effective approach to achieve a smooth drive. LIM operation does not require the use of Permanent Magnets (PM), thus avoiding the increased weight and cost of the MAGLEV system. The available publications on LIM are very diverse, and most are written for a specific LIM with specific operations. It is reasonable to believe that a two-dimensional analysis will yield better results than his one-dimensional theory when calculating the performance of a motor. Analyze LIM efficiency using a maglev train drive system. This system has the highest energy consumption. This is because the efficiency of LIM increases as the slip frequency decreases. In this article, the components of secondary current density and air gap magnetic flux density are used to study the performance and characteristics of levitation and propulsion forces. The analysis is performed by his two-dimensional application of the field system using two models, with and without iron base [1-10].

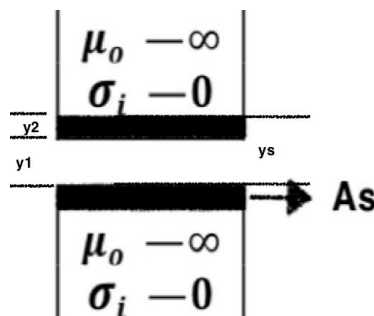
Two-dimensional model with back-iron

In this work, the study is interested in two-dimensional field analysis because the air gap field has both y and x flux components. If x flow component is present, a non-constant in the y direction is required to satisfy the condition

$$\text{Div } \vec{B} = 0.$$

Variation of the y-component of the air gap field along the air gap length including a conductive secondary sheet metal line. In addition to current shift phenomena in secondary leaves. The two-dimensional magnetic field analysis is performed using a magnetic vector potential that is a rotation of B. This allows us to consider the effects of physical air gaps and finite magnetic permeability and conductivity in different regions in the y direction. A model for two-dimensional field analysis of a steel-based machine is shown in Figure 1. The coordinate axes are selected as shown.

Figure 1. Two-Regions model for the two-dimensional analysis.



Region 1:  $0 \leq y \leq y1$ , is the air gap region between the stator and the secondary.

Region 2:  $y1 \leq y \leq ys$  the secondary conductive layer  $ys = y1 + tc$  In this idealized model, the effects of hysteresis and saturation are also ignored, and the stator has infinite permeability and conductivity will be zero.

In this model, a smooth surface stator with continuous current plates and an electric stator load (As) is used instead of a slotted stator.

$$A_s(x,t) = \text{Re} \sum_m A_{sm} e^{j(\omega t - \alpha x)} \dots\dots (1)$$

The current flowing in the z direction corresponds to the stator current. It is only assumed that the secondary current is still flowing in the axial z direction. As a result, the magnetic flux density B and the magnetic field strength H both have x and y components. The electromagnetic theory approach is based on Maxwell's field equations for an isotropic medium.

$$\text{Cur } 1 \vec{H} = \vec{J}, \quad \text{Cur } 1 \vec{E} = \frac{\partial \vec{B}}{\partial t} + \text{Cur } 1 (\vec{v} \times \vec{B}) \dots\dots (2a, b)$$

They are complemented by the continuity conditions

$$\text{Div } \vec{J} = 0, \quad \text{div } \vec{B} = 0 \dots\dots (3a, b)$$

and the relation between the magnetic field intensity and flux density.

$$\vec{B} = \mu \vec{H} \dots\dots (3c)$$

The current density is obtained by

$$\vec{J} = \sigma \vec{E} \dots\dots (4)$$

The magnetic vector potential  $\vec{V}$  is defined by

$$\vec{B} = \text{Cur } 1 \vec{V} \dots\dots (5a)$$

with the complementary condition

$$\text{div } \vec{V} = 0 \dots\dots (5b)$$

and the magnetic Reynold's number r is

$$r = W \mu_0 \frac{\sigma}{\dots} \dots\dots (5c)$$

In the air-gap region, with zero conductivity

$$\frac{\partial^2 V_{z1}}{\partial y^2} - a^2 V_{z1}(y) = 0 \dots\dots (6)$$

By resolving the differential equations (5C) and (6) in accordance with the problem's geometry, the magnetic vector potentials are derived. The complex amplitudes of the vector are obtained from the solution of these differential equations.

$$V_{z2} = C_2 e^{ky} + D_2 e^{-ky} \dots\dots (7)$$

$$V_{z1} = C_1 e^{ay} + D_1 e^{-ay} \dots\dots (8)$$

Where C and D are the integration constants which will be determined by applying the boundary conditions.

**The boundary conditions**

Applying the following boundary conditions will allow us to evaluate the unknown integration constants C and D:

The tangential component of the field intensity  $H_x$  is continuous across each interregional boundary.

$$H_x = \frac{1}{\mu} \frac{\partial V_z}{\partial y} \dots\dots (9a)$$

We have, at the boundary between regions n and n + 1,

$$\frac{1}{\mu} \frac{\partial V_z}{\partial y} = \frac{1}{\mu+1} \frac{\partial V_{z+1}}{\partial y} \dots\dots (9b)$$

At the first air gap stator boundary, the condition becomes modified by introducing the exciting stator electric loading

$$\frac{1}{\mu} \frac{\partial V_{z1}}{\partial y} |_{y=0} = A_s, \dots\dots (10a)$$

Since  $\mu_i$  tends to infinity.

At the secondary air gap-core, boundary yields.

$$\frac{1}{\mu} \frac{\partial V_{z2}}{\partial y} |_{y=y_s} = 0 \dots\dots (10b)$$

The same condition is to be applied at the air gap-secondary boundaries yielding,

$$\frac{1}{\mu} \frac{\partial V_{Z1}}{\partial y} \Big|_{y=y1} = \frac{1}{\mu} \frac{\partial V_{Z2}}{\partial y} \Big|_{y=y1} \dots\dots (10c)$$

The continuity of the vector potential ( $V_{zn} = V_{zn} + 1$ ) is implied by the fact that the normal component of the flux density  $B_y$  is continuous across each interregional boundary.

When the stipulation is used at the secondary-air gap border,

$$V_{Z1} \Big|_{y=y1} = V_{Z2} \Big|_{y=y1} \dots\dots (10d)$$

To fully determine the integration constants P and Q, the conditions in equations (10a-d) must be met. Hence, the vector potential for the two regions and the flux density components can be completely obtained.

Applications of the boundary conditions gives the four integration constants of the two regions. Thus, the flux density components due to the stator electric loading in the different machine regions are

$$B_{x1} = a(C_1 e^{ay} - D_1 e^{-ay}) \dots\dots (11a)$$

$$B_{y1} = j a(C_1 e^{ay} - D_1 e^{-ay}) \dots\dots (11b)$$

$$B_{x2} = k (C_2 e^{ky} - D_2 e^{-ky}) \dots\dots\dots (11c)$$

$$B_{y2} = j a (C_2 e^{ky} - D_2 e^{-ky}) \dots\dots\dots (11d)$$

**Simulation results of phase angle control for LIM with iron backing**

The driven force is function of ( $B_y, J_z$ ) and the levitation force is function of ( $B_x, J_z$ ), as the amplitudes of :-  $F_y = B_x \cdot J_z$  and  $F_x = B_y \cdot J_z$ .

From the following graphs Figure 2. (Levitation Force with iron backing with the speed  $v$  [m/s]) and Figure 3 (drive force with iron backing with the speed  $v$  [m/s]) the relations between the flux density in the x-direction and speed is illustrated decreasing the flux density with increasing the speed.

**Figure 2.** Levitation force with iron backing with the speed  $v$  [m/s].

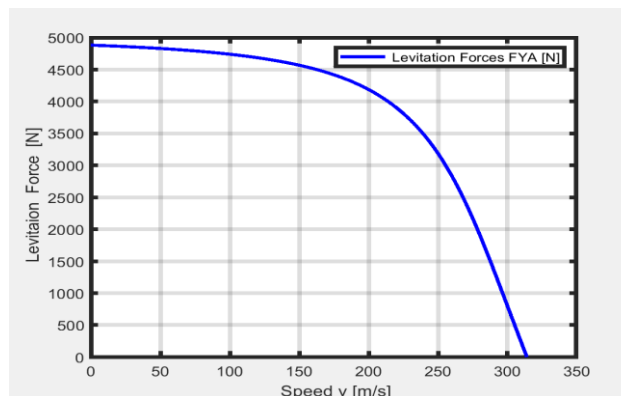
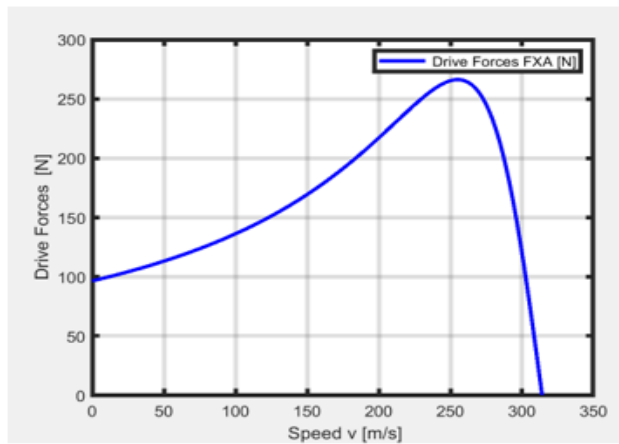


Figure 3. Drive force with iron backing with the speed  $v$  [m/s].



**The Two-Dimensional field analysis without iron backing**

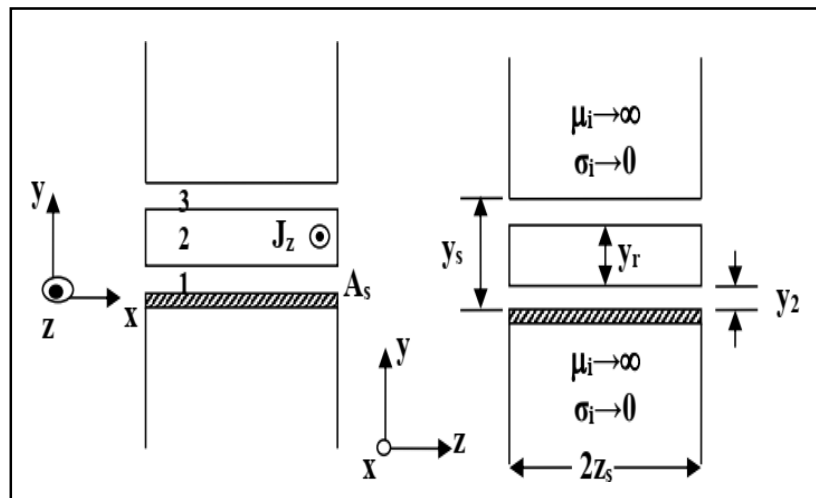
The three regions of the model in Figure.4 are defined as follows:

Region 1:  $0 \leq y \leq y_2$ , is the first air-gap between the external stator and the secondary sheet.

Region 2:  $y_2 \leq y \leq (y_2 + y_r)$ , is the secondary sheet conducting layer.

Region 3:  $(y_2 + y_r) \leq y \leq y_s$ , the second air-gap between the secondary sheet layer and the iron core.

Figure 4. Two-dimensional analysis without iron backing (three regions models).



**Simulation of phase angle control for LIM without iron backing**

The following graphs (Figures 5-9) show many parameters (Current Density, Levitation Flux Density ( $B_x$ ), Drive Flux Density ( $B_x$ ), Levitation Force (N) and Drive Force (N)) with the speed  $v$  [m/s]. as clear how speed and the flux density in the  $x$ -direction relate to one another, with speed decreasing the flux density.

Figure 5. Current density  $J_z$  with the speed  $v$  [m/s].

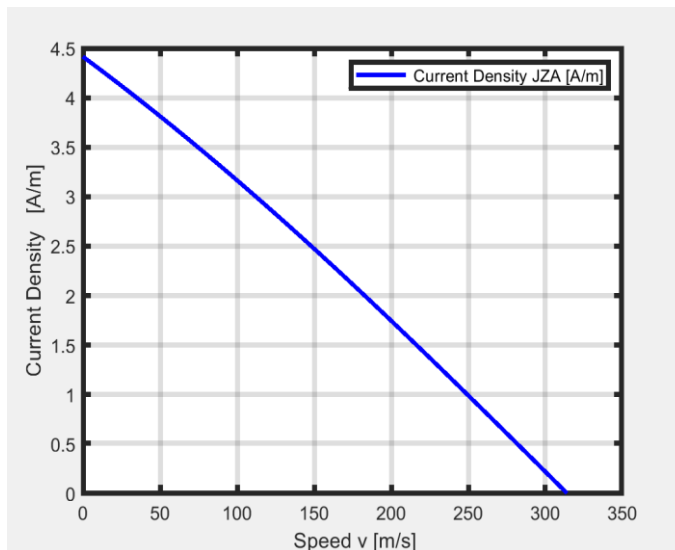


Figure 6. Levitation flux density ( $b_x$ ) with the speed  $v$  [m/s].

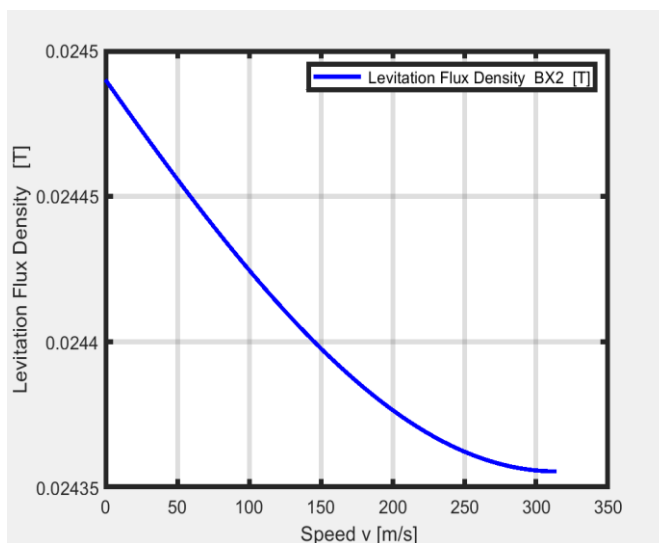


Figure 7. Drive flux density ( $b_x$ ) and the speed  $v$  [m/s].

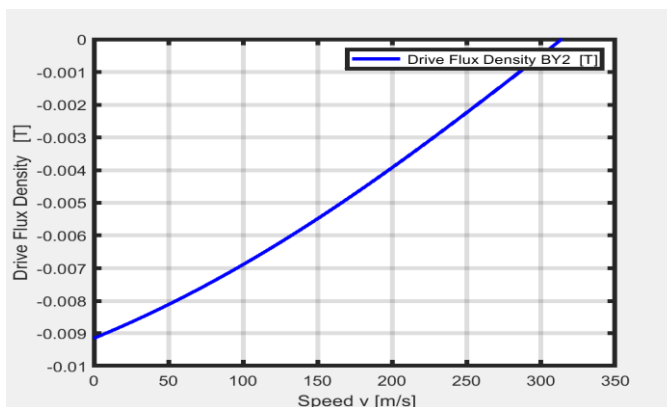


Figure 8. Levitation force (N) and the speed  $v$  [m/s].

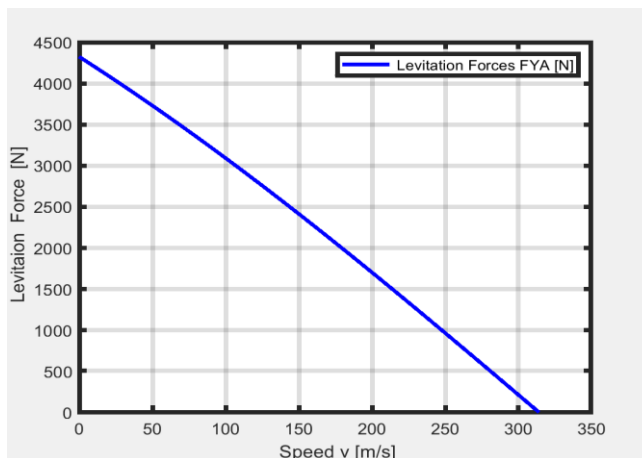
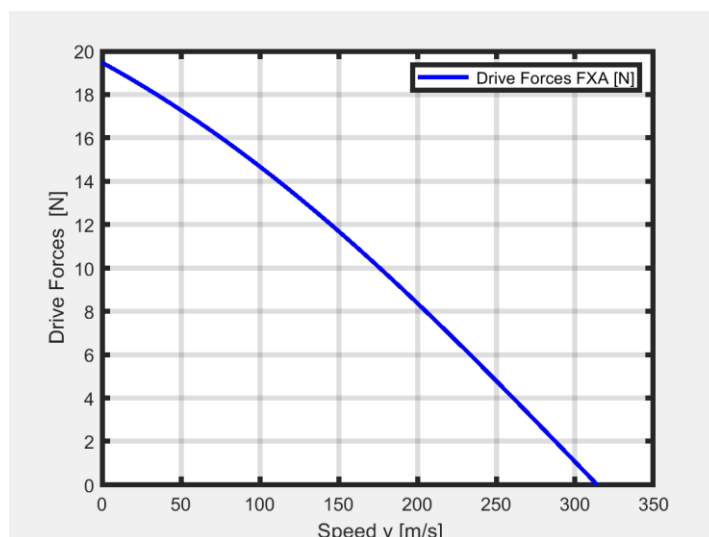


Figure 9. Drive force (N) and the speed  $v$  [m/s].



### Comparison between forces with backing and without iron backing

As illustrated below in Figures 10-13, the relations between (Drive and Levitation forces with iron backing and without iron backing in different cases)

Figure 10. Drive and levitation forces without iron backing with the speed  $v$  [m/s].

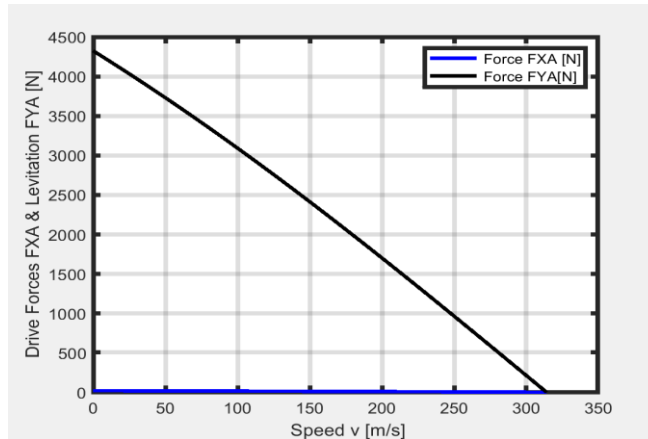


Figure 11. Drive forces with and without iron backing with the speed  $v$  [m/s].

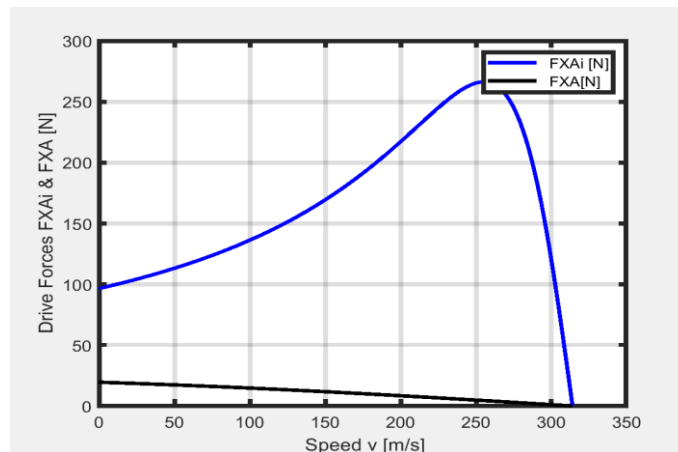


Figure 12. Levitation force with and without iron backing with the speed  $v$  [m/s].

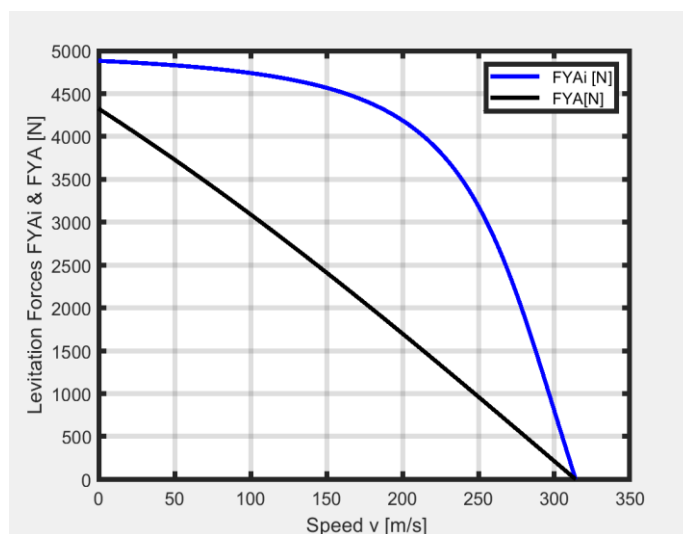
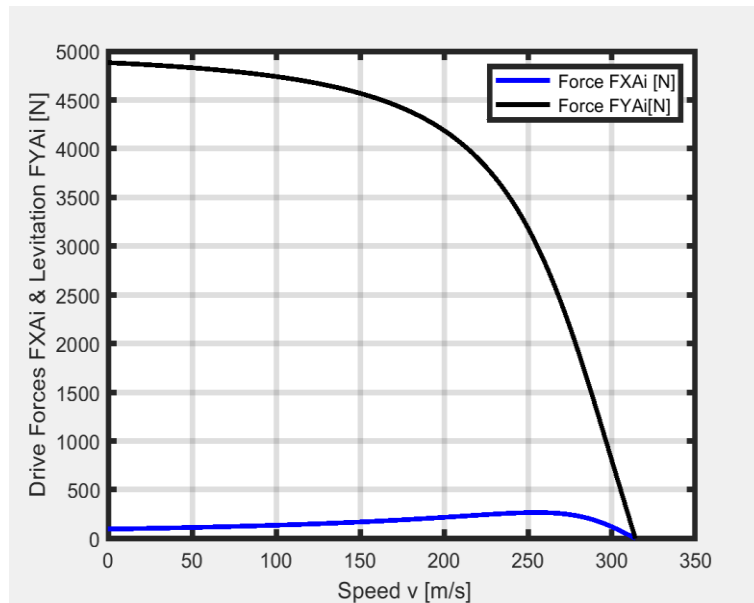




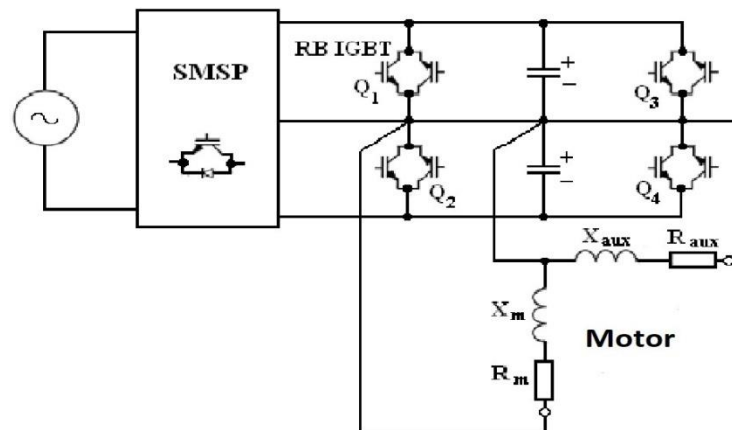
Figure 13. Drive force and levitation force with iron backing with the speed  $v$  [m/s].



**Control with IGBT**

Using the phase angle value between the two phases voltage source to control the output characteristics of the motor:

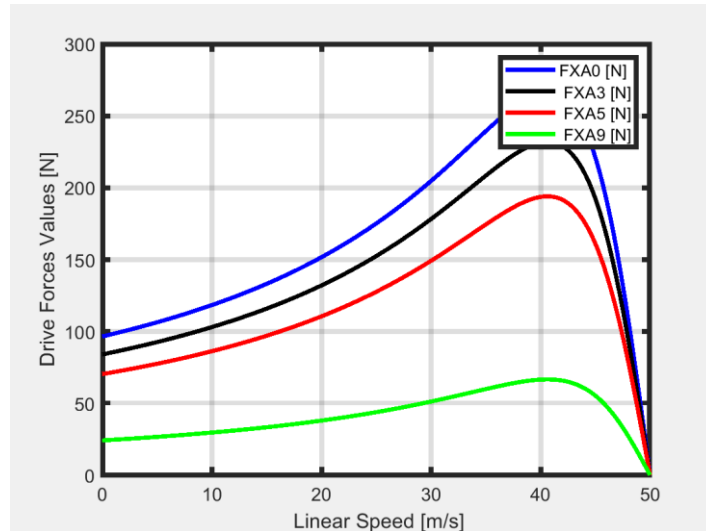
Figure 14. Phase angle control circuit diagram of two-phase induction motor [11-20].



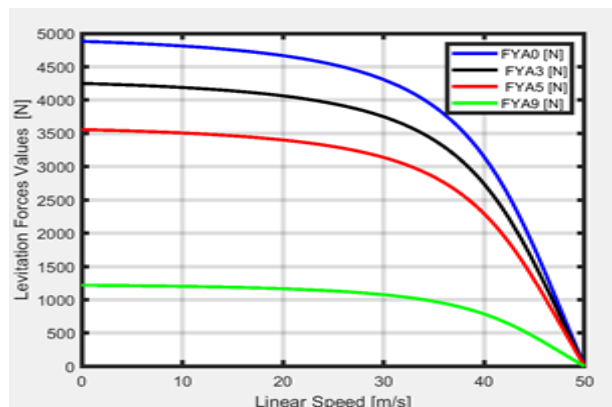
For the phase angle control at four values of this angle using the iron backing case in some control cases.

As in the following figures (Figures 15-18).

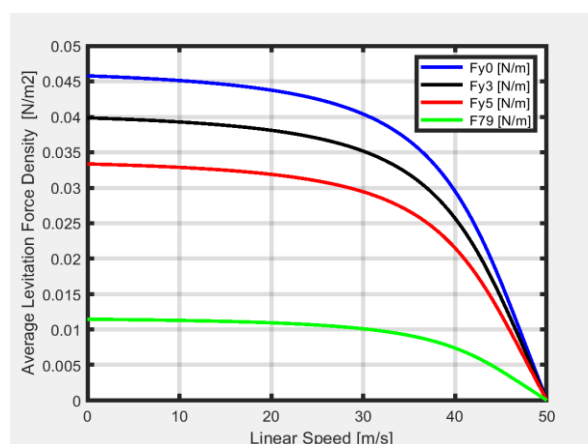
**Figure 15.** Drive force at different values of phase angle between the two phases with the linear speed [m/s] considered the control angles are (0°, 30°, 45° and 90°).



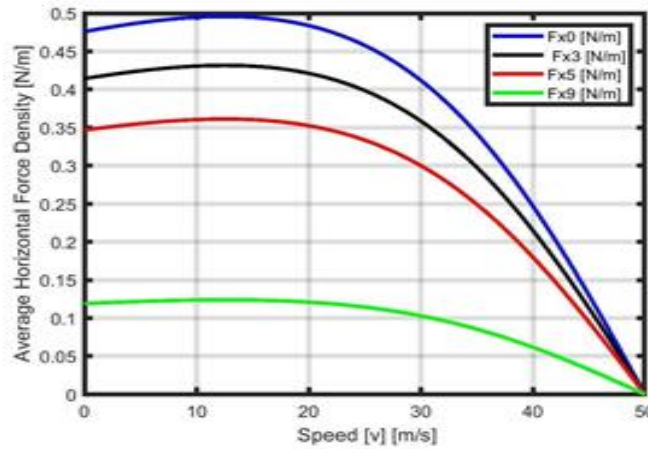
**Figure 16.** Levitation force at different values of phase angle between the two phases with the linear speed [m/s] Considered the control angles are (0°, 30°, 45° and 90°).



**Figure 17.** Average levitation force Density [N/m<sup>2</sup>] at different values of phase angle between the two phases with the Linear Speed [m/s] considered the control angles are (0°, 30°, 45° and 90°).



**Figure 18.** Average Drive force density [N/m<sup>2</sup>] at different values of phase angle between the two phases with the linear Speed [m/s] considered the control angles are (0°, 30°, 45° and 90°).



### RESULTS AND DISCUSSION

From all the previous figures, a large force value is obtained when the control angle is 0° (the phase angle between the two phases is 90°), so the phase angle is used to control the propulsion and levitation forces and can control the values of these forces at different values of controlled angles [21-30]. The magnetic field analysis in Cartesian coordinates is performed based on a two-domain model starting from Maxwell's magnetic field equations. Two models were selected.

- With secondary iron support (with internal air gap area and conductive secondary area).
- No secondary iron-back (with inner air gap region, outer air gap region, and conductive secondary region).

From the point of view of electromagnetic theory, this approach is based on Maxwell's field equations for an isotropic medium moving with velocity  $\mathbf{v}^*$ , to obtain differential equations describing the vector potential in different regions. Solve the field equations using boundary conditions for each case. The unknown integration constants C and D are evaluated by applying appropriate boundary conditions. The air gap flux density and secondary current density components due to the stator electrical load are expressed using two-dimensional analysis. Air gap flux density and secondary current density distributions can be easily calculated and graphed using a variety of models. These calculations are done using mechanical design data. The normalized components of radial by and tangential flux density Bx are plotted against engine speed. Models with iron supports on the stator and secondary of the motor have a higher radial flux density than models without iron supports. The secondary current density has a maximum value at standstill, and as the speed increases, the secondary current density decreases. This is the main effect of armature reaction force. Two components of force characteristics (propulsive force and levitation force) are determined and examined for different models. Demonstrate phase shift control by varying the phase shift angle from 0 to 90°, and compare its characteristics [31-43].

### CONCLUSION

In this paper, we have extensively examined the functionality of Linear Induction Motors (LIM) featuring back iron components. Our focus has been on elucidating a comprehensive two-dimensional model to effectively compute both force and flux density within the system. This meticulous model stands as a reliable alternative to the conventional finite element method, offering enhanced accuracy and efficiency in calculations pertaining to LIMs. By delving into the intricacies of LIMs with back iron, the aim is to provide analytical toolset for optimizing motor performance and design. Through rigorous analysis and validation, it establishes the viability and efficacy of this two-dimensional model, paving the way for its practical implementation in diverse applications.

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