ISSN (Print) : 2320 – 3765
ISSN (Online): 2278 – 8875

International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 1, January 2015

Designing of TOU Power Price Based on Game-Theoretic Approach

K.R.Akshara, Prof. P.Sangameshwara Raju

Department of EEE, SV University, Tirupati, AP, India

Department of EEE, SV University, Tirupati, AP, India

ABSTRACT: Requirement for power differs throughout the day, improving the average cost of power source. A time-of-use (TOU) expense has been suggested as a demand-side management (DSM) technique to influence customer requirements. In this paper, we explain a game-theoretic approach to improve TOU expenses strategies (GT-TOU). We recommend designs of expenses to power organizations coming up from customer demand fluctuations, and designs of customer fulfillment with the difference between the affordable demand and the actual intake.

KEYWORDS*:* Electricity price, game theory, optimization, smart grid, time-of-use.

I. INTRODUCTION

The fluctuation of energy demand throughout the day has long been a problem for energy companies. During peak time, the energy companies encounter significant pressure to provide clients with enough h energy, and may even have to ration the energy resource of certain locations when the gap between demand and development is too large. During off-peak time, only a few of generators are needed to offer sufficient electricity to meet up with client need, and the nonproductive generators result in a invest of development prospective. The system complete is not the highest possible complete that a system can offer, but operate far from base load is not cost efficient, and may harm the stability of the Time-of-use (TOU) costs is an efficient technique of demand-side management (DSM) that application companies can implement to influence customer actions. Hart way et al. confirmed experimentally that TOU is profitable to a application organization, and that in common, the customers are satisfied with the TOU cost choice. California's Statewide Pricing Lead revealed that personal and small-to-medium commercial and professional clients are willing to reduce their peak-period power use due to time-varying pricing.

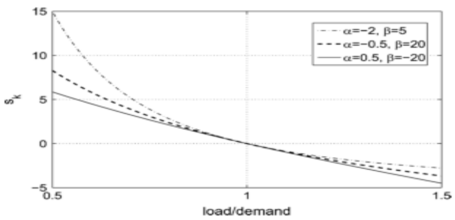


Fig1. s_k With different parameters α_k and β_k (with unit d_k).

GAME MODEL FOR SINGLE USER TYPE

In this part we prepare the representation with a single type of user. We divide a day into N periods, where depends on the situation of the application. For hourly-based pricing, the notations are planned below.

C_k: marginal cost of electricity
p_k: unit sales price of electricity
g_k: electricity generation
d_k: nominal user demand
l_k: actual user load in response to the price

ISSN (Print) : 2320 – 3765
ISSN (Online): 2278 – 8875

International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 1, January 2015

The subscript $k = 1,...,N$ denotes the equivalent time period. For simplicity, we also use the notations P= {[p_1, p2,…………,p_N]^T

And $\{p_k\}_{k=1}^{N}$ in this document. We model the profit of the company as

P=$\sum_{k=1}^{N} p_k l_k - \sum_{k=1}^{N} c_k g_k - f(g)$ (1)

Where $f(g)$ corresponds to the cost caused by the difference of customer command during the day. We model this cost using the sum of squared generation deviations from the mean, multiplied by

A coefficient μ, i.e.

f(g)=$\mu\sum_{k=1}^{N}(gk - \bar{g})^2$ (2)

Where $\bar{g}$ is the common power production during the day. The cost function C of electricity users includes the money they pay for the electricity and their satisfaction with the service, i.e.

C= $\sum_{k=1}^{N} p_k l_k + \sum_{k=1}^{N} s_k(l_k, d_k)$ (3)

Where $s_k(l_k, d_k)$ Signifies the customer contract operates. The loss of the function value, however, decelerates as the real fill is constantly on the improve, because the customers will not be "infinitely" more satisfied when they use more power. When the real fill is equal to the user requirement, the operate value is zero. Therefore the satisfaction function $s_k(l_k, d_k)$ should fulfill the following conditions:

1) *If* $l_k = d_k$, $s_k(l_k, d_k)= 0$

2) *If* $l_k > d_k$, $s_k(l_k, d_k) < 0$, $\frac{\partial sk}{\partial lk} < 0$, $\frac{\partial^2 s_k}{\partial {l_k}^2} > 0$

3) *If* $l_k < d_k$,

These conditions are related to the conditions for service functions projected in, but are dissimilar because the conditions here are used to model the satisfaction with the difference between demand and actual load. In this document we select $s_k(l_k, d_k)$ as

$$s_k(l_k, d_k) = d_k \beta_k [(\frac{l_k}{d_k})\wedge\alpha k - 1] \quad (4)$$

Where $\alpha_k = 1$ *and* $\alpha_k\beta_k < 0$ This function satisfies all three conditions listed above. An design of with dissimilar parameters and is shown in Fig. 1. From the example we can see that by adjusting the parameters and, (4) can be used to characterize dissimilar types of users. Therefore the service function of the company is its profit minus the satisfaction cost of the users, i.e.

$$\mu_1 = \sum_{k=1}^{N} p_k l_k - \sum_{k=1}^{N} c_k g_k - \sum_{k=1}^{N} s_k(l_k, d_k) - f(g) \quad (5)$$

The utility function of users is the negative of the cost function, i.e.,

$$\mu 2 = -c = -\sum_{k=1}^{N} p_k l_k - \sum_{k=1}^{N} sk(lk, dk) \quad (6)$$

The objective is to maximize the usefulness functions u_1 and u_2 under certain constraints. The optimization problem is formulated as functions μ1 and μ2 under particular constraints.

$$(p^*, g^*) = \underset{p,g}{\arg max}$$

$$u_1 = \sum_{k=1}^{N} (p_k \ l_k - c_k \ g_k - s_k) - f(g) \quad (7)$$

$$l^* = \underset{l}{\arg max}$$

$$u_2 = -\sum_{k=1}^{N}(p_k l_k + s_k) \quad (8)$$

Subject to $l_{k,min} \le l_k \le g_k \le min \ (d_{k,max,gk,max})$,

$$c_k \le p_k, \quad k = 1,2, \dots . N.$$

The restrictions are used to control the action of the utility company and the customers. To ensure the minimum ad $l_{k,\min}$ required by customers, Also, the real load cannot exceed, which is the lowest $\{d_{k,\max}, g_{\max}\}$ between the maximum possible customer load $d_{k,\max}$ at period k of time and the maximum generation limit $g_{k,\max}$. In actual power systems, the total generation should match the user load at all times, which is controlled by the system operator.

ISSN (Print) : 2320 – 3765
ISSN (Online): 2278 – 8875

International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 1, January 2015

Therefore we can simplify the problem by letting $l_k = g_k$ .Let $l_{k,\max} = \min\{d_{k,\max}, g_{k,\max}\}$, the problem can then be rewritten as

$$(p^*, g^*) = \arg\max_{p}$$

$$u_1 = \sum_{k=1}^{N} (p_k\ l_k - c_k\ l_k - s_k) - f(l) \qquad (9)$$

$$l^* = \arg\max_{l}$$

$$u_2 = -\sum_{k=1}^{N} (p_k l_k + s_k) \qquad (10)$$

Subject to $l_{k,min} \le l_k \le l_{k,max,}$

K=1, 2….N,

$$c_k \le p_k, \qquad k = 1,2, \dots. N.$$

In this game model, the service companies decide the TOU Price P, and the electrical energy users decide the actual utilization of electricity L according to the price. Let L denote the approach set of the utility company, which are all the possible TOU prices the company can set. Let L denote the strategy set of the users, the strategy sets can be defined as follows:

$$p = \{p | p \in \mathbb{R}^N, l_{min} \le l(p) \le l_{max,p} \ge c\}$$

$$\mathcal{L} = \{l | l \in \mathbb{R}^N, l_{min} \le l \le l_{max}\}$$

Note that in the definition of P we write L as a function of P, because the actual user load is dependent on the prices. We aim to find the best possible price $\boldsymbol{p}^* \in \mathcal{P}$ and best possible load response $\boldsymbol{l}^* \in \mathcal{L}$ such that Nash balance $(\boldsymbol{p}^*, \boldsymbol{l}^*) \in \mathcal{P} \times \mathcal{L}$ is achieved between the service company and electrical energy users. A strategy profile is called Nash equilibrium if any one-sided change of strategy by a single agent does not increase

Ts utility function

$$\forall_p \in \mathcal{P}, p \ne p^* : u_1(p^*, l^*) \ge u_1(p, l^*)$$

$$\forall_l \in \mathcal{L}, l \ne l^* : u_2(p^*, l^*) \ge u_2(p^*, l)$$

II. OPTIMIZING UTILITY FUNCTIONS

Since this is a multi-stage activity, we use in reverse induction to fix for the stability. The application organization requires action first by establishing the power cost, and then clients adjust the quantity of power they use. Therefore, according to the in reverse introduction concept, we first maximize with respect to $\{l_k\}_{k=1}^{N}$, and then connect the maximum fill response $\boldsymbol{l}^*(\boldsymbol{p})$ into and improve with regard to $\{p_k\}_{k=1}^{N}$.

A. Optimal Demand Response to Price

In organize to find a user's optimal demand response to the cost set by the helpfulness company, we consider the electrical energy prices of different time periods $\{p_k\}_{k=1}^{N}$ as given, and take the first-order Derivatives of u_2 with respect to$\{p_k\}_{k=1}^{N}$:

$$\frac{\partial_{u2}}{\partial_{lk}} = -p_k - \alpha_k \beta_k \left(\frac{l_k}{d_k}\right)^{\alpha_k - 1} \qquad (11)$$

Above equations equal to zero then

$$l_k^* = \left(-\frac{p_k}{\alpha_k \beta_k}\right)^{1/(\alpha_k - 1) d_k} \qquad (12)$$

Second order derivative of utility function u2 is

$$\frac{\partial_{u2}^2}{\partial l_k \partial l_i} = \begin{cases} -\alpha_k \beta_k (\alpha_k - 1) \dfrac{l_k^{\alpha_k - 2}}{d_k^{\alpha_k - 1}} \\ 0 \end{cases} \qquad (13)$$

When k = i

When k≠I Since $\alpha_k < 1$ and $\alpha_k \beta_k < 0$, the diagonal elements of the Hessian matrix are all negative, and the off-diagonal elements are all zero. The Hessian matrix is negative definite, meaning that $\{l_k^*\}_{k=1}^{N}$ is the optimal user load given P price .Let

ISSN (Print) : 2320 – 3765
ISSN (Online): 2278 – 8875

International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering
(An ISO 3297: 2007 Certified Organization)
Vol. 4, Issue 1, January 2015

$\epsilon_k = \frac{1}{\alpha_k - 1} < 0$, $k = 1,2, \ldots . N$ (14) And
$\eta_k = -\alpha_k \beta_k > 0$, $k = 1,2, \ldots N$ (15)
We can then rewrite (12) as

$$l_k^* = (\frac{p_k}{\eta_k})^{\epsilon_k} \quad (16)$$

For ease of notation, we will use (16) instead of (12) in the rest of this document .We need to simplify that the most favorable response l_k^* is not always in the form of (16), if a dissimilar contentment function is chosen based on user characteristics.

B. Optimal price Based on User Response:
We will increase the utility function of organizations by finding the maximum costs strategy based on the customer reaction. Connecting (16) into (5), we obtain as a operate of P as follows:

$$u_1(p) = \sum_{k=1}^{N} \{p_k l_k^*(p_k) - c_k l_k^*(p_k) - s_k[l_k^*(p_k), d_k]\} - f[l^*(p)] \quad (17)$$

Given the optimal user load as a function of the electricity price, we can rewrite the constraints on user loads as constraints on the prices. From (16) we obtain

$$p_k = \left(\frac{l_k}{d_k}\right)^{1/\epsilon_k} \eta_k \quad (18)$$

Since (18) is a decreasing function of l_k, the constraints on prices can be written as

$$p_{k,min} \le p_k \le p_{k,max} \quad (19)$$

Where $p_{k,min} = max\{c_k, (\frac{l_{k,max}}{d_k})^{\frac{1}{\epsilon_k}} \eta_k\}$

$$p_{k,max} = (l_{k,min} / d_k)^{\frac{1}{\epsilon_k}} \eta_k$$

The optimization of with respect to the Prices p now becomes

$$\max_{p} \; u_1(p)$$

The restrictions of this marketing issue are straight line. To ensure that the remedy is the best possible, the negative-definiteness of the Hessian matrix of u_1 is parameter dependent. In a traditional TOU costs technique, a day is separated into several blocks of hours, and each block is regarded as "peak", "semi-peak", or "off-peak" time. The cost is continuous in every time block. An intuitive presentation of the restriction is to implement constant $p_i = p_j$ price within each prevent by including straight line restrictions of the form, when i and j are time periods in the same block. The constraint can be published as

$$\mathbf{A} \cdot \mathbf{p} = \mathbf{0} \quad (20)$$

Where A is an n*n matrix with $A_{i,j} \in \{-1, 0, 1\}$ and $\mathbf{A} \cdot$1=0 Here 1 denotes an all-one vector of dimension, N *1 and 0 and denotes an all-zero vector of dimension N*1. Assume we choose N =24 and set the starting of each hour as the start of that time period. matrix is set to be the matrix shown in Fig. 2(a) shown in Fig2(b). In this case a day is divided into four time blocks with three different price levels.

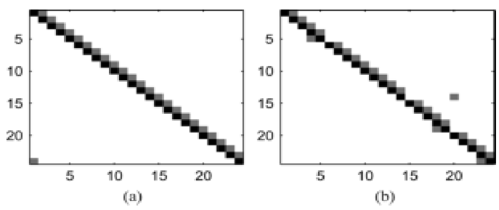


Fig.2. Illustration of the matrix. Black denotes 1, light grey denotes and white denotes 0. (a) Matrix A for flat pricing. (b) Matrix A for TOU blocks pricing.

ISSN (Print) : 2320 – 3765
ISSN (Online): 2278 – 8875

International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 1, January 2015

III. MODEL WITH MULTIPLE USER TYPES

We consider three types of users: residential users (R), commercial users (B), and small industrial users (F). These users have different price response characteristics.

Residential users:

Users in places are usually delicate to price change, and they would like to modify their intake of electricity according to the time-varying costs. The flexibility of residential customers is relatively low, as they have limited capability to decrease or improve their complete use of power.

Commercial users:

During office time, the requirements for power in business districts are great, and professional customers do not want to decrease the use of power which may impact their business. Power preservation techniques, however, can be used to preserve power if the power cost is great. Aspect of the less time-urgent perform can also be planned to other times of day.

Industrial users:

Commercial customers, especially those with great power consumption facilities, use a lot of power. The objective is to stage the complete load of all customers instead of just one type of customer. The organization and each kind of customer would have a utility function reflecting its overall profit/cost. The application functions are detailed below, where u_1 is for the application company; u_{2R}, u_{2B}, and u_{2F} are for personal, professional, and professional customers, respectively:

$$u_1 = \sum_{k=1}^{N}(pR_k lR_k + pB_k lB_k + pF_k lF_k) - \sum_{k=1}^{N} c_k l_k - \sum_{k=1}^{N}(sR_k + sB_k + sF_k) - f(l) \qquad (21)$$
$$u_{2R} = -\sum_{k=1}^{N} pR_k lR_k - \sum_{k=1}^{N} sR_k \qquad (22)$$
$$u_{2B} = -\sum_{k=1}^{N} pB_k lB_k - \sum_{k=1}^{N} sB_k \qquad (23)$$
$$u_{2R} = -\sum_{k=1}^{N} pF_k lF_k - \sum_{k=1}^{N} sF_k \qquad (24)$$

Here L_k is the sum of the loads of all types of users, i.e.

$$l_k = l_{Rk} + l_{Bk} + l_{Fk}, \quad k = 1,2,\ldots.N \qquad (25)$$

And $f(l)$ is the price due to fluctuation of complete customer loads and are fulfillment features for personal, commercial and commercial customers, respectively. These features have different factors based on the characteristics of the users, and may take different kinds other than (4) if necessary. In this project, we implement the same fulfillment operate as in the single-user-type situation with different factors, and the users' optimal reactions to costs are in a kind just like (16), with different parameters ϵ and η . The maximum reactions of different types of customers can be acquired as follows:

$$l_{Rk}^* = \left(\frac{p_{Rk}^*}{\eta_{Rk}}\right)^{\epsilon_{Rk}} d_{Rk} \qquad (26)$$
$$l_{Bk}^* = \left(\frac{p_{Bk}^*}{\eta_{Bk}}\right)^{\epsilon_{Rk}} d_{Bk} \qquad (27)$$
$$l_{Fk}^* = \left(\frac{p_{Fk}^*}{\eta_{Fk}}\right)^{\epsilon_{Rk}} d_{Fk} \qquad (28)$$

We then find the optimal prices by solving the optimization problem similar to (20) as follows:

$$\min_{p_R, p_B, p_F} u_1(p_R, p_B, p_F)$$
$$\begin{aligned} p_{R,min} &\le p_R \le p_{R,max} \\ p_{B,min} &\le p_B \le p_{B,max} \\ p_{F,min} &\le p_F \le p_{F,max} \end{aligned} \qquad (29)$$

Future scope:

The proposed framework can be extended in a number of ways to consider more detailed physical effects and market design structures such as transmission constraints and coupled day-ahead and real-time markets (two-settlement markets). The model can also be constructed with more realistic set-ups where the suppliers bid their operational information and the ISO **(International Organization for Standardization)** clears the market by solving a unit

ISSN (Print) : 2320 – 3765
ISSN (Online): 2278 – 8875

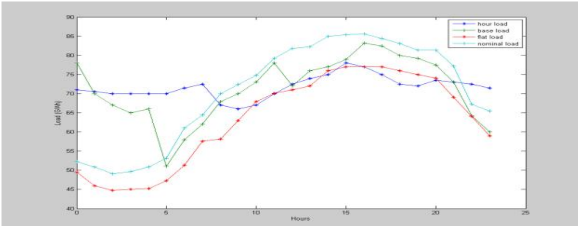
International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering

(An ISO 3297: 2007 Certified Organization)

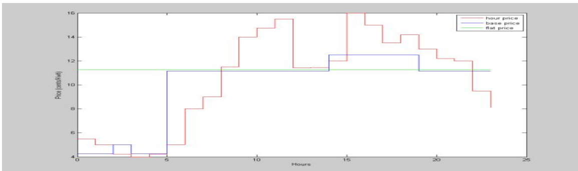
Vol. 4, Issue 1, January 2015

commitment problem in the day-ahead market and an economic dispatch model in the real-time market. Other settings include information exchange, cooperation, and use of forecasting capabilities by the suppliers.

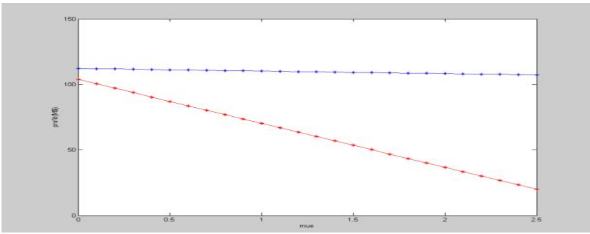
IV. SIMULATION RESULTS



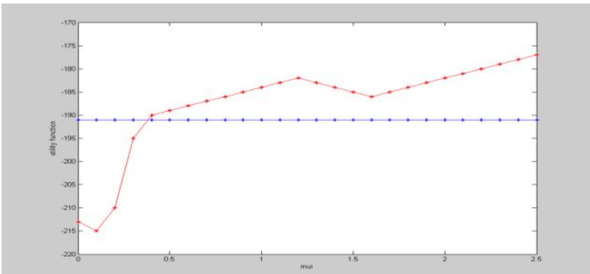
Comparison of loads between flat pricing and GT-TOU pricing.



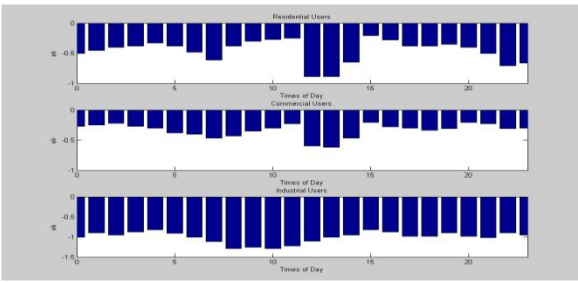
Comparison of prices and GT-TOU pricing.



Comparison of company profit and GT-TOU pricing as changes.



Comparison of user benefit the flat pricing and GT-TOU pricing as changes.



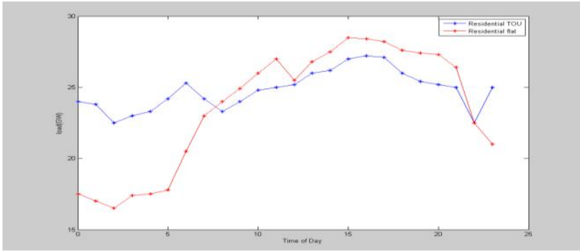
Parameter ϵ_k for different types of users used in the numerical examples

ISSN (Print) : 2320 – 3765
ISSN (Online): 2278 – 8875

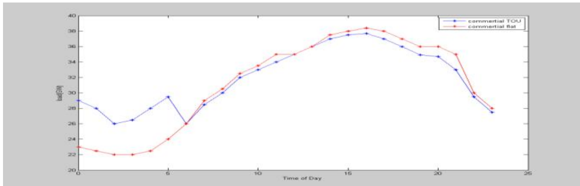
International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering

(An ISO 3297: 2007 Certified Organization)

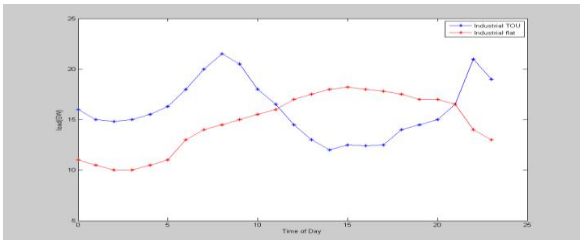
Vol. 4, Issue 1, January 2015

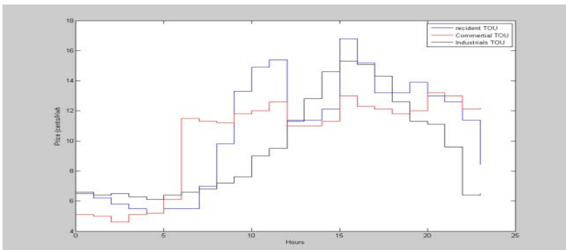


Residential users before and after applying GT-TOU Pricing



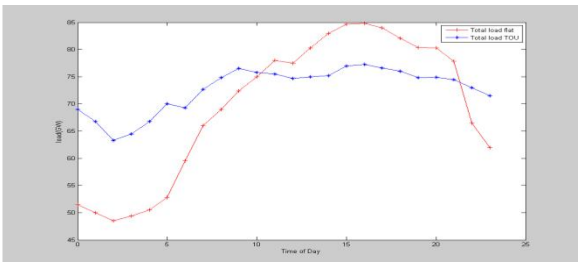
Commercial users before and after applying GT-TOU Pricing





Industrial users before and after applying GT-TOU Pricing

Comparison of GT-TOU prices with flat prices for multiple user types.



Comparison of total load before and after GT-TOU pricing.

ISSN (Print) : 2320 – 3765
ISSN (Online): 2278 – 8875

International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 1, January 2015

V. CONCLUSIONS

We suggested a maximum game-theoretic TOU electricity pricing technique (GT-TOU). We developed application functions for both power organizations and customers, and fixed for a Nash equilibrium, which provides maximum costs and customer reactions. The pricing strategy is flexible, as the design is appropriate for several pricing patterns, such as on per hour basis costs and time-block TOU costs. Simulation results illustrate that our approach can level user demand, raise the profits of the service companies, and decrease component prices for electrical energy users, and make sure overall user benefit. The leveled user load also potentially helps make sure a more steady power system.

REFERENCES

[1]C. W. Gellings, "The concept of demand-side management for electric utilities," *Proc. IEEE*, vol. 73, pp. 1468–1470, 1985.

[2] S.Rahman and SR.Rinaldy, "Anefficient load model for analyzing demand side management impacts," *IEEE Trans. Powe rSyst.*,vol.8, pp. 1219–1226, 1993.

[3] M. Fahrioglu and F. L. Alvarado, "Designing cost effective demand management contracts using game theory," in *Proc .IEEEPower Engineering Society 1999 Winter Meeting*, New York, Jan. 1999, pp.427–432.

[4] M. Fahrioglu and F. L. Alvarado, "Designing incentive compatible contracts for effective demand management," *IEE E Trans. Power Syst.*,vol. 15, pp. 1255–1260, 2000.

[5]D.W.CavesandL.R.Christensen,"Econometric analysis of residential time-of-use electricity pricing experiments," *J. Econometr.*, vol.14, pp. 287–306, 1980.