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A Brief Note on Geometry and Topology in Mathematics

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Editorial

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EDITORIAL NOTE

Geometry

Geometry and topology is a general term for the historically distinct disciplines of geometry and topology in mathematics, because general frameworks allow both disciplines to be manipulated uniformly, most notably in Riemannian geometry's local to global theorems and results like the Gauss–Bonnet theorem and Chern-Weil theory. It's not to be confused with "geometric topology," which focuses on topological applications to geometry. It contains the following items:

- Topology and differential geometry.
- Topology in terms of geometry (including low-dimensional topology and surgery theory).

Although some sections of geometry and topology (such as surgery theory, particularly algebraic surgery theory) are highly algebraic, it does not contain aspects of algebraic topology such as homotopy theory.

Geometry has local (or infinitesimal) structure, but topology has only global structure. Geometry, on the other hand, has continuous moduli, whereas topology has discrete moduli [1].

By way of example, Riemannian geometry is an example of geometry, whereas homotopy theory is an example of topology. Geometry is the study of metric spaces, while topology is the study of topological spaces.

The phrases aren't often used interchangeably: symplectic manifolds are a boundary case, and coarse geometry is global rather than local.

Differentiable manifolds of a fixed dimension are all locally diffeomorphic to Euclidean space by definition; hence there are no local invariants other than dimension. Differentiable structures on a manifold are hence topological [2-4]. A Riemannian manifold's curvature, on the other hand, is a local (indeed, infinitesimal) invariant (and is the only local invariant under isometry).

Moduli

A structure is considered to be rigid if it contains discrete moduli (if it has no deformations or if a deformation of a structure is isomorphic to the original structure), and topology is the study of it (whether it is a geometric or topological structure). The structure is considered to be flexible if it exhibits non-trivial deformations, and its study involves geometry [5].

Because the space of homotopy classes of maps is discontinuous, topology is the study of maps up to homotopy. Differentiable structures on a manifold are typically discrete spaces, and hence examples of topology, although unusual R4s feature continuous moduli of differentiable structures. Because algebraic varieties have continuous moduli spaces, algebraic geometry is the study of them. These are moduli spaces with finite dimensions.

On a particular differentiable manifold, the space of Riemannian metrics is infinite-dimensional.

Symplectic manifolds

Symplectic manifolds are a boundary case, and symplectic topology and symplectic geometry are two aspects of their study [6-8]. A symplectic manifold has no local structure, according to Darboux's theorem; hence their study is called topology. The space of symplectic structures on a manifold, on the other hand, forms continuous moduli, suggesting that their study be dubbed geometry. However, the space of symplectic structures is discrete up to isotopy (any family of symplectic structures are isotopic).

Geometric topology is the study of manifolds and their mappings, particularly embeddings of one manifold into another, in mathematics.

The distinction between geometric and algebraic topology may be traced back to Reidemeister torsion's 1935 categorization of lens spaces, which necessitated separating spaces that are homotopy equivalent but not homeomorphic [9]. Simple homotopy theory was born out of this. The term "geometric topology" appears to have just lately been used to characterise them.

Manifolds of dimension 5 and above, or embeddings in similarities 3 and above, are referred to as high-dimensional topology. Low-dimensional topology is concerned with issues of up to four dimensions, or embeddings of up to two dimensions.

Dimension 4 is unique in that it is high-dimensional in some ways (topologically), but low-dimensional in others (differentially); this overlap results in phenomena that are unique to dimension 4, such as exotic differentiable structures on R4. Thus, topological categorization of 4-manifolds is straightforward in principle, and the crucial considerations are whether a topological manifold admits a differentiable structure and, if so, how many. Notably, the smooth case of dimension 4 is the generalised Poincare conjecture's last open case.

The difference is made because surgery theory operates in dimensions 5 and higher (in fact, it works topologically in dimension 4, but proving this is difficult), and so surgery theory controls the behaviour of manifolds in these dimensions algebraically [10]. Surgery theory does not operate in dimensions 4 and lower (topologically, in dimensions 3 and below), and other events arise.

Because the Whitney embedding theorem, the main technical innovation that underpins surgery theory, needs 2+1 dimensions, the difference at dimension 5 is exact. The Whitney trick, in general, allows one to "unknot" knotted spheres–or, more precisely, remove self-intersections of immersions; it accomplishes this by using a homotopy of a disc–the disc has two dimensions, and the homotopy adds one more–and thus in codimensions greater than two, this can be done without intersecting itself; thus embeddings in codimension greater than two can be understood by surgery. In surgical theory, the essential step occurs in the middle dimension, therefore the Whitney method works when the middle dimension has a codimension more than 2 (roughly, 212 is adequate, so total dimension 5 is enough) [11].

Casson handles are a four-dimensional version of the Whitney trick: since there aren't enough dimensions, a Whitney disc produces additional kinks, which can be resolved by another Whitney disc, resulting in a succession ("tower") of discs. Surgery works topologically but not differently ably in dimension 4 because the tower's limit gives a topological but not differentiable map [12].

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