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Excess Covariance and the "Hole Problem" in General Relativity

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Commentary

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ABOUT THE STUDY

In an essay on Einstein's heuristics, I argued that the sole basis for a covariance requirement in the general theory of relativity is Einstein's "principle of equivalence" as it pertains to arbitrarily moving rigid frames in finite Galilean regions [1]. Particularly decisive is the accommodation of rotating coordinates. This is in contrast to the usual view according to which general relativity must be generally covariant because it admits of non-Euclidean spacetimes [2]. However, the employment of a non-Euclidean metric has no direct connection to general covariance since the latter pertains to different coordinate representations of the same spacetime, not to the representation of different spacetimes. The covariance group singled out by the principle of equivalence is a sub-group of the general covariance group and so Einstein's generally covariant gravitational field law carries what we could call "Excess Covariance." Thus while general covariance does indeed carry heuristic force in Einstein's theory of gravity, that heuristic force is indirect, by virtue of general covariance encompassing the group of coordinate transformations corresponding to arbitrary rigid frames as per the principle of equivalence.

I suggest here that the notion of excess covariance also can shed light on the so called "Hole Problem" in general relativity, the failure of the field equation to uniquely determine the gravitational field at a point. In a well-known paper [3,4], John Earman and John Norton launched a philosophical debate, which continues in the literature, on whether the Hole Problem precludes a "Substantivalist" understanding of space and time. According to Earman and Norton, spacetime substantivalism, or the view that spaciotemporal points exist in their own right, so to speak, whether or not occupied by matter, renders general relativity an indeterministic theory since we cannot decide which of all possible solutions is the one physically realized at a point.

The awkwardness of this result, however lies not merely in indeterminism per se but in that the multiple solutions leave all observables unaltered, as if the same gravitational field simply were "dragged" from one location in space and time to another. Einstein himself in 1915 resolved the hole problem, at least to his own satisfaction, with a philosophical argument denying the independent existence of points of space and time apart from the gravitational field. On that account the multiple solutions are distinct merely mathematically, but not physically (thus "Diffeomorphism Equivalence").

But Einstein's solution leaves unresolved the question of the physical nature of the gravitational field. Is the gravitational field physically real in the sense of having a definite value at each point in space and time? If so, how are we to regard the problem of mathematical under determination? Or should we regard the field merely as a mathematical model and embrace some version of relationalism as regards space and time? This obviates the Hole Problem, but arguably at the cost of a significant lessening of physical insight into the nature of gravity. How can spacetime be "curved" or exhibit a non-Euclidean metric if the gravitational field is not physically real?

Hole indeterminism is often characterized as a "gauge freedom" of the general theory of relativity, which strikes me as rather grandiose given that the mathematical under determination at issues arises simply through the excess covariance of the Ricci tensor (that is, in the pure field equation $R_{\mu\nu} = 0$ the generalized field equation need to be brought in for our purposes). For were there available a differential operator with less than general covariance, but still accommodating arbitrary motions of rigid frames in finite Galilean regions and therefore with no effect on the conceptual structure of the general theory of relativity then the hole problem would not arise in the first place. Indeed [5]. We can trace the hole problem to the very employment of a differentiable manifold in the representation of the gravitational field. For with the manifold we posit at the same time a set of coordinatized points in space and time, but initially without the metric structure furnished by the gravitational field. The manifold is in this sense an abstract or symbolic space, not a physical space. And this very lack of metric structure engenders the possibility of the Hole Problem in a generally covariant theory. The Hole Problem and its associated indeterminism is merely an artifact of excess covariance and has no interesting implications for the question of spacetime substantivalism *versus* relationalism.

Ultimately, the Hole Problem is a legacy of Descartes' introduction of numerical coordinates in his analytical geometry of 1637. Einstein observes in a 1934 essay that Descartes' use of numbers to identify geometrical points essentially coins the modern mathematical concept of "absolute space", since these coordinated points are at rest by definition, as it were [6]. The inertia-determining properties of Cartesian space thus served an indispensable function in Newtonian physics. It is an irony that the very attempt by Einstein to overcome absolute background structure through general covariance in his theory of gravity leads to the Hole Problem, a vestige of Cartesian coordinate space.

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