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Critical Droplets and Spin Glass Ground States

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Commentary

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ABOUT THE STUDY

Spin glasses are a class of disordered magnetic materials that present a unique ongoing challenge to statistical physics. Their characteristic behaviours—spins freezing in random orientations at low temperature, a cusp in the magnetic susceptibility at the freezing temperature unaccompanied by any observable singularity in the specific heat, extremely slow relaxation and equilibration and dynamical memory/aging effects—were first observed in certain dilute magnetic alloys (such as a few percent iron atoms randomly distributed in a gold lattice), but have since been observed in other classes of materials [1].

The difficulty in analysing spin glasses theoretically arises from the simultaneous presence of quenched randomness and frustration. Unlike more conventional magnetic systems, in a spin glass the spin-spin interactions can be either ferromagnetic or antiferromagnetic, which are quenched (i.e., fixed) and randomly distributed throughout the material so that the interaction between an arbitrarily chosen pair of spins has roughly equal a priori probability of being either ferromagnetic or antiferromagnetic. This leads to a property, known as frustration, wherein no configuration of the spins can satisfy all of the interactions [2-4].

A measure of difficulty of spin glasses is that it took a decade to solve and understand a mean field version. Typically such models are (comparatively) easy to solve, but in the mean field spin glass the nature of symmetry breaking known today as replica symmetry breaking was so unexpected, complex and exotic that the 2021 Nobel Prize in Physics was awarded to Giorgio Parisi, who solved the mean field problem through discovering the correct nature of its broken symmetry [5-10].

While mean field theories typically do not provide an accurate description of the critical behavior of a low-dimensional system, they are generally an excellent guide to the nature of symmetry breaking and the system order parameter at low temperatures. However, in the spin glass the nature of the low-temperature phase in dimensions above two remains open. While some in the field believe replica symmetry breaking indeed describes the low-temperature behavior in three dimensions and higher, others suspect that the correct description is very different, in particular, an alternative picture known as droplet-scaling differs sharply from replica symmetry breaking [11-15].

One of these differences concerns the nature and number of ground states in the thermodynamic limit. For simplicity, we consider the Ising spin glass in which each spin can take on only one of two values, say plus or minus one. Ground states are defined as infinite-volume spin configurations whose energy cannot be lowered by flipping any finite set of spins; clearly in the Ising model with only pairwise interactions and in the absence of a magnetic field ground states come in globally spin-reversed pairs. Replica symmetry breaking predicts that an infinite spin glass will have infinitely many distinct ground state pairs, no one of which can be transformed into any other by any simple global symmetry transformation. Droplet-scaling, on the other hand, predicts only a single spin-reversed pair of ground states. As of now, the answer is unknown in any dimension greater than one though there are partial results in two dimensions that indicate only a single pair of ground states.

One way of approaching ground state behavior is to examine ground state stability, which given the unique features of a spin glass can be thought of in different ways. One is to ask what happens to a particular ground state when all of the spin-spin interactions or simply, couplings are independently changed by an infinitesimally small amount [13,15-17]. A more recent approach is to ask what happens when a single coupling is changed by an arbitrary amount, with all other couplings fixed [18-22].

In the latter approach every coupling will have a single critical value in which a droplet (i.e., a connected set of spins) flips, with the boundary of the droplet, which is defined on the dual lattice passing through the edge whose coupling was varied. This droplet is called the critical droplet (for that particular edge in that particular ground state), and it was shown in [23], as well as [22], that the distribution of critical droplet geometries and energies carries information that can determine which (if any) of the various low-temperature scenarios is correct in a given dimension.

It was further shown that the predictions of replica symmetry breaking for ground states is equivalent to the presence of a positive density of edges in any ground state whose critical droplets have boundaries comprising a positive density of all edges in the lattice. This reduces one of the fundamental questions in spin glass physics to one of stability and critical droplet behavior. In particular, it shows that if replica symmetry breaking is correct, the ground state is only marginally stable in the sense that varying a single coupling can cause a global (infinite) change in a ground state [23].

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