

# NUMERICAL SOLUTION FOR BOUNDARY VALUE PROBLEM USING FINITE DIFFERENCE METHOD

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**Abstract:** In this paper, Numerical Methods for solving ordinary differential equations, beginning with basic techniques of finite difference methods for linear boundary value problem is investigated. Numerical solution is found for the boundary value problem using finite difference method and the results are compared with analytical solution. MATLAB coding is developed for the finite difference method. The results are reported for conclusion.

**KEYWORDS:** Ordinary Differential Equations, finite Difference method, Boundary value problem, Analytical solution, Numerical solution

## I. INTRODUCTION

In mathematics, finite-difference methods are numerical methods for approximating the solutions to differential equations using finite difference equations to approximate derivatives. Finite differences method is used in soil physics problems. An important application of finite differences is in numerical analysis, especially in numerical differential equations, which aim at the numerical solution of ordinary and partial differential equations respectively. The idea is to replace the derivatives appearing in the differential equation by finite differences that approximate them. The resulting methods are called finite difference methods. Common applications of the finite difference method are in computational science and engineering disciplines, such as thermal engineering, fluid mechanics, etc.

### A. STEPS INVOLVED IN FINITE DIFFERENCE METHOD

A finite difference method typically involves the following steps: Generate a grid, for example  $(x_i; t(k))$ , where we want to find an approximate solution.

- ❖ Substitute the derivatives in a system of ordinary differential equations with finite difference schemes.
- ❖ The ordinary differential equation then becomes a linear/non-linear system of algebraic equations.
- ❖ Solve the system of algebraic equations. Implement and debug the computer code.
- ❖ Do the error analysis, both analytically and numerically.

### B. DERIVATION OF FINITE DIFFERENCE METHOD

Let consider the linear equations

$$x'' = p(t)x'(t) + q(t)x(t) + r(t) \quad (1)$$

With boundary condition  $x(a) = \alpha$  and  $x(b) = \beta$ .

To start the derivation first replace each term  $x(t_j) = x_j$

$$x_j'' = p_j x_j' + q_j x_j + r_j \quad (2)$$

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And the formula for central difference formula for first derivative

$$\text{Gives, substitute } x(t_j) = x_j \text{ in the above equation, and get } x'_j = \frac{x_{j+1} - x_{j-1}}{2h} + o(h^2) \quad (3)$$

Now consider the second derivative of central difference formula equation

$$x''(t_j) = \frac{x(t_{j+1}) - 2x(t_j) + x(t_{j-1}))}{h^2} + o(h^2) \quad (4)$$

Substitute  $x(t_j) = x_j$  in equation (4) and get

$$x''_j = \frac{x_{j+1} - 2x_j + x_{j-1}}{h^2} + o(h^2) \quad (5)$$

Substitute equation (3) and (5) in (2) get,

$$\frac{x_{j+1} - 2x_j + x_{j-1}}{h^2} + o(h^2) = p(t_j) \left( \frac{x_{j+1} - x_{j-1}}{2h} + o(h^2) \right) + q(t_j)x_j + r(t_j) \quad (6)$$

Next, drop the two terms in  $o(h^2)$  in equation (6)

$$\frac{x_{j+1} - 2x_j + x_{j-1}}{h^2} + o(h^2) = p_j \left( \frac{x_{j+1} - x_{j-1}}{2h} \right) + q_j x_j + r_j \quad (7)$$

And introduce the notation  $p_j = p(t_j)$ ,  $q_j = q(t_j)$  and  $r_j = r(t_j)$  in equation (7).

This produces the difference equation

$$\frac{x_{j+1} - 2x_j + x_{j-1}}{h^2} = p_j \left( \frac{x_{j+1} - x_{j-1}}{2h} \right) + q_j x_j + r_j \quad (8)$$

this is used to compute numerical approximation to the differential equation. This is carried out by multiplying each side of (8) by  $h^2$ , and then collecting terms involving  $x_{j-1}$ ,  $x_j$  and  $x_{j+1}$  and arraying them in a system of linear equations.

(7) multiply by  $h^2$  gives

$$\begin{aligned} \left( \frac{x_{j+1} - 2x_j + x_{j-1}}{h^2} \right) h^2 &= h^2 \left[ p_j \left( \frac{x_{j+1} - x_{j-1}}{2h} \right) + q_j x_j + r_j \right] \\ x_{j+1} - 2x_j + x_{j-1} - \frac{h}{2} p_j x_{j+1} + \frac{h}{2} p_j x_{j-1} - h^2 q_j x_j &= h^2 r_j \end{aligned} \quad (9)$$

equation (9) multiply by -1

$$-x_{j+1} + 2x_j - x_{j-1} + \frac{h}{2} p_j x_{j+1} - \frac{h}{2} p_j x_{j-1} + h^2 q_j x_j = -h^2 r_j$$

Now collecting the term  $x_{j-1}$ ,  $x_j$  and  $x_{j+1}$

$$\left( -\frac{h}{2} p_j - 1 \right) x_{j-1} + (2 + h^2 q_j) x_j + \left( \frac{h}{2} p_j - 1 \right) x_{j+1} = -h^2 r_j \quad (10)$$

for  $j = 1, 2, \dots, N-1$  in equation (10). when  $j = 1$

$$\left( -\frac{h}{2} p_1 - 1 \right) x_0 + (2 + h^2 q_1) x_1 + \left( \frac{h}{2} p_1 - 1 \right) x_2 = -h^2 r_1 \quad (11)$$

And  $x_0 = \alpha$  the equation (11) becomes

$$\begin{aligned} \left( -\frac{h}{2} p_1 - 1 \right) \alpha + (2 + h^2 q_1) x_1 + \left( \frac{h}{2} p_1 - 1 \right) x_2 &= -h^2 r_1 \\ (2 + h^2 q_1) x_1 + \left( \frac{h}{2} p_1 - 1 \right) x_2 &= -h^2 r_1 + \left( \frac{h}{2} p_1 + 1 \right) \alpha \\ (2 + h^2 q_1) x_1 + \left( \frac{h}{2} p_1 - 1 \right) x_2 &= -h^2 r_1 + e_o \end{aligned} \quad (12)$$

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where  $e_0 = (\frac{h}{2} p_1 + 1)\alpha$

when  $j = N-1$

$$(-\frac{h}{2} p_{N-1} - 1)x_{N-2} + (2 + h^2 q_{N-1})x_{N-1} + (\frac{h}{2} p_{N-1} - 1)x_N = -h^2 r_{N-1}$$

And  $x_N = \beta$ , then the equation (13) becomes

$$\begin{aligned} (-\frac{h}{2} p_{N-1} - 1)x_{N-2} + (2 + h^2 q_{N-1})x_{N-1} &= -h^2 r_{N-1} - [\frac{h}{2} p_{N-1} - 1]\beta \\ (-\frac{h}{2} p_{N-1} - 1)x_{N-2} + (2 + h^2 q_{N-1})x_{N-1} &= -h^2 r_{N-1} + [1 - \frac{h}{2} p_{N-1}]\beta \\ &= -h^2 r_{N-1} + e_N \\ (-\frac{h}{2} p_{N-1} - 1)x_{N-2} + (2 + h^2 q_{N-1})x_{N-1} &= -h^2 r_{N-1} + e_N \end{aligned}$$

(13)

where  $e_N = [1 - \frac{h}{2} p_{N-1}]\beta$

The system in equation (10), (12), (13) shows how the familiar triangle is formed, which is more visible when displayed with matrix notations

$$\begin{bmatrix} 2+h^2q_1 & \frac{h}{2}p_1-1 & 0 & 0 & \dots & 0 & \dots & 0 \\ -\frac{h}{2}p_2-1 & 2+h^2q_2 & \frac{h}{2}p_2-1 & 0 & \dots & 0 & \dots & 0 \\ 0 & -\frac{h}{2}p_3-1 & 2+h^2q_3 & \frac{h}{2}p_3-1 & \dots & 0 & \dots & 0 \\ & & \vdots & & & & & \\ 0 & 0 & -\frac{h}{2}p_j-1 & 2+h^2q_j & \frac{h}{2}p_j-1 & \dots & & 0 \\ 0 & 0 & 0 & -\frac{h}{2}p_{N-2}-1 & 2+h^2q_{N-2} & \frac{h}{2}p_{N-2}-1 & & 0 \\ 0 & 0 & 0 & 0 & \dots & \dots & -\frac{h}{2}p_{N-1}-1 & 2+h^2q_{N-1} \end{bmatrix} \times \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_j \\ \vdots \\ x_{N-2} \\ x_{N-1} \end{bmatrix} = \begin{bmatrix} -h^2r_1 + e_0 \\ -h^2r_2 \\ -h^2r_3 \\ \vdots \\ -h^2r_j \\ \vdots \\ -h^2r_{N-2} \\ -h^2r_{N-1} + e_N \end{bmatrix}$$

(14)

where  $e_0 = (\frac{h}{2} p_1 + 1)\alpha$  and  $e_N = (\frac{-h}{2} p_{N-1} + 1)\beta$

## II. FINITE DIFFERENCE METHOD FOR SOLVING BOUNDARY VALUE PROBLEM

$x''(t) = \frac{2t}{1+t^2} x'(t) - \frac{2}{1+t^2} x(t) + 1$  with  $x(0) = 1.25$  and  $x(4) = -0.95$  over the interval  $[0,4]$ .

### A. NUMERICAL SOLUTION FOR BOUNDARY VALUE PROBLEM

The given equation is

$$x''(t) = \frac{2t}{1+t^2} x'(t) - \frac{2}{1+t^2} x(t) + 1 \tag{15}$$

And consider the linear equation

$$x_j'' = p_j x_j' + q_j x_j + r_j \tag{16}$$

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comparing equations (15) and (16) the values of  $p_j, q_j, r_j$  are obtained. Approximation solution for boundary value problem using finite difference method.

$$\left(-\frac{h}{2} p_j - 1\right)x_{j-1} + (2 + h^2 q_j)x_j + \left(\frac{h}{2} p_j - 1\right)x_{j+1} = -h^2 r_j$$

The finite-difference method is used to construct numerical solutions  $\{x_j\}$  using the system of equations (10). There are 41 terms in the sequence generated with  $h_2 = 0.1$ , and the sequence  $\{x_{j,2}\}$  only includes every other term from these computations; they correspond to the 21 values of  $\{t_j\}$  given in **Table 1**. Similarly, the sequences  $\{x_{j,3}\}$  and  $\{x_{j,4}\}$  are a portion of the values generated with step sizes  $h_3 = 0.05$  and  $h_4 = 0.025$ , respectively, and they correspond to the 21 values of  $\{t_j\}$  in **Table 1**.

### B. ANALYTIC (OR) EXACT SOLUTION FOR THE BOUNDARY VALUE PROBLEM

Next compare numerical solutions in **Table 1** with the analytic solution. Let consider the equation (15), integrating twice with respect to  $t$  to the limit 0 to 4.

$$x(t) = 1.25 + 0.486089652t - 2.25t^2 + 2t \arctan(t) - \frac{1}{2} \ln(1+t^2) + t \frac{1}{2} \ln(1+t) \tag{17}$$

Put  $t=0, x(0)=1.25000$ . Continuing this process the values are presented in the below table. Comparing the values of  $x(t)$ , in **Table 1** and **Table 2**. The numerical solutions have error of order  $o(h^2)$ .

### C. ERRORS IN NUMERICAL APPROXIMATION USING FINITE DIFFERENCE METHOD

Hence reducing the step size by a factor of  $\frac{1}{2}$  results in the error being reduced by about  $\frac{1}{4}$ . For instance, at  $t_j = 1.0$  the errors

incurred with step sizes  $h_1=0.2, h_2=0.1, h_3=0.05$ , and  $h_4=0.025$  are  
 $e_{j,1} = x(t_j) - x_{j,1} = 1.056886 - 1.042106 = 0.014780, e_{j,1} = 0.014780$   
 $e_{j,2} = x(t_j)_{\text{exact}} - x_{j,2} = 1.056886 - 1.053226 = 0.003660, e_{j,2} = 0.003660$   
 $e_{j,3} = x(t_j) - x_{j,3} = 1.056886 - 1.055973 = 0.000913, e_{j,3} = 0.000913$  and  
 $e_{j,4} = x(t_j) - x_{j,4} = 1.056886 - 1.056658 = 0.000228, e_{j,4} = 0.000228$

Their successive ratios

$$\frac{e_{j,2}}{e_{j,1}} = \frac{0.003660}{0.014780} = 0.2476 \quad \frac{e_{j,3}}{e_{j,2}} = \frac{0.000913}{0.003660} = 0.2495 \quad \text{and} \quad \frac{e_{j,4}}{e_{j,3}} = \frac{0.000228}{0.000913} = 0.2497$$

are approaching  $\frac{1}{4}$ . A careful scrutiny of **Table 3** will reveal that this is happening. And **FIGURE-I** shows that the difference between the numerical approximation solution and analytic solution.

### D. RICHARDSON'S IMPROVEMENT SCHEME

Richardson's improvement scheme can be used to extrapolate the seemingly inaccurate sequences  $\{x_{j,1}\}, \{x_{j,2}\}, \{x_{j,3}\}$ , and  $\{x_{j,4}\}$  and obtain six digits of precision. Eliminate the error terms  $o(h^2)$  and  $o((h/2)^2)$  in the approximations  $\{x_{j,1}\}$  and  $\{x_{j,2}\}$  by generating the extrapolated sequence

$$\{z_{j,1}\} = \{(4x_{j,2} - x_{j,1}) / 3\}. \tag{18}$$

Similarly, the error terms  $o((h/2)^2)$  and  $o((h/4)^2)$  for  $\{x_{j,2}\}$  and  $\{x_{j,3}\}$  are eliminated by generating

$$\{z_{j,2}\} = \{(4x_{j,3} - x_{j,2}) / 3\} \tag{19}$$

It has been shown that the second level of Richardson's improvement scheme applies to the sequences  $\{z_{j,1}\}$  and  $\{z_{j,2}\}$  so the third improvement is

$$= \{(16z_{j,2} - z_{j,1}) / 15\} \tag{20}$$

Now illustrate the situation by finding the extrapolated values that correspond to  $t_j = 1.0$ . The first extrapolated value is

$$\frac{4x_{j,2} - x_{j,1}}{3} = \frac{4(1.053226) - 1.042106}{3} = 1.056932 = z_{j,1} \tag{21}$$

The second extrapolated value is

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$$\frac{4x_{j,3} - x_{j,2}}{3} = \frac{4(1.055973) - 1.053226}{3} = 1.056889 = z_{j,2} \quad (22)$$

Finally, the third extrapolation involves the terms  $z_{j,1}$  and  $z_{j,2}$ :

$$\frac{16z_{j,2} - z_{j,1}}{15} = \frac{16(1.056889) - 1.056932}{15} = 1.056886 \quad (23)$$

This last computation contains six decimal places of accuracy. The values at the other points are given in **Table 4**.

### E. MATLAB PROGRAM FOR FINITE DIFFERENCE METHOD USING SCRIPT FILE

```

clc;
% problem definition
aa=0;
bb=4;
n=20;
alpha=1.25;beta=-0.95;
h=(bb-aa)/n;
t=zeros(1,n+1);
x=zeros(1,n-1);
a=zeros(1,n-2);
b=zeros(1,n-1);
c=zeros(1,n-2);
d=zeros(1,n-1);
t=aa+h:h:aa+h*(n-1);
p=(2*t/(1+t.^2)).*ones(1,n-1);
q=(-2/(1+t.^2)).*ones(1,n-1);
r=1.*ones(1,n-1);
% end problem definition
x=linspace(aa+h,bb,n);
a=zeros(1,n-1);
a(1:n-2)=-1-p(1,1:n-2)*h2;
d=(2+hh*q);
b=zeros(1,n-1);
b(2:n-1)=-1+p(1,2:n-1)*h2;
c(1)=hh*r(1)+(1+p(1)*h2)*alpha;
c(2:n-2)=-hh*r(2:n-2);
c(n-1)=hh*r(n-1)+(1-p(n-1)*h2)*beta;
x=trimat(a,d,b,c);
tt=[aa t bb];
xx=[alpha x beta];
out=[tt' xx'];
disp(out)plot(tt,xx)
grid on
    
```

### F. MATLAB program for solving Tridiagonal systems using function file program

```

function x=trimat(A,D,C,B)
%Input- A is the sub diagonal of the co-efficient matrix
%      - D is the main diagonal of the co-efficient matrix
    
```

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```

% - C is the super diagonal of the co-efficient matrix
% - B is the constant vector of the linear system
%Output- x is the solution vector
N=length(B);
for k=2:N
    mult=A(k-1)/D(k-1)
    D(k)=D(k)-mult*C(k-1);
    B(k)=B(k)-mult*B(k-1);
end
x(N)=B(N)/D(N);
for k=N-1:-1:1
    x(k)=(B(k)-C(k)*x(k+1))/D(k);
end
    
```

**TABLE I  
NUMERICAL APPROXIMATION FOR**

$$x''(t) = \frac{2t}{1+t^2} x'(t) - \frac{2}{1+t^2} x(t) + 1$$

$t_j$	$x_{j,1}$ $h_1=0.2$	$x_{j,2}$ $h_2=0.1$	$x_{j,3}$ $h_3=0.05$	$x_{j,4}$ $h_4=0.025$
0.0	1.250000	1.250000	1.250000	1.250000
0.2	1.314503	1.316640	1.317174	1.317306
0.4	1.320607	1.325045	1.326141	1.326414
0.6	1.272755	1.279533	1.281206	1.281623
0.8	1.177399	1.1896438	1.188670	1.189227
1.0	1.042106	1.053226	1.055973	1.056658
1.2	0.874878	0.887823	0.891023	0.891821
1.4	0.683712	0.698181	0.701758	0.702650
1.6	0.476372	0.494027	0.495900	0.496865
1.8	0.260264	0.276749	0.280828	0.281846
2.0	0.042399	0.059343	0.063537	0.064583
2.2	-0.170616	-0.153592	-0.149378	-0.148327
2.4	-0.372557	-0.355841	-0.351702	-0.350669
2.6	-0.557565	-0.541546	-0.537580	-0.536590
2.8	-0.720114	-0.705188	-0.701492	-0.700570
3.0	-0.854988	-0.841551	-0.838223	-0.837393
3.2	-0.957250	-0.945700	-0.942839	-0.942145
3.4	-1.022221	-1.012958	-1.010662	-1.010090
3.6	-1.045457	-1.038880	-1.037250	-1.036844
3.8	-1.022727	-1.019238	-1.018373	-1.018158
4.0	-0.950000	-0.950000	-0.950000	-0.950000

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**TABLE II  
EXACT SOLUTION FOR THE GIVEN BOUNDARY VALUE PROBLEM**

$t_j$	$x(t_j)$
0.0	1.2500
0.2	1.317350
0.4	1.326505
0.6	1.28762
0.8	1.189412
1.0	1.056886
1.2	0.892086
1.4	0.702947
1.6	0.497187
1.8	0.282184
2.0	0.064931
2.2	-0.147977
2.4	-0.350325
2.6	-0.536261
2.8	-0.700262
3.0	-0.837116
3.2	-0.941888
3.4	-1.009899
3.6	-1.036709
3.8	-1.018086
4.0	-0.950000

**TABLE III  
ERRORS IN NUMERICAL APPROXIMATIONS USING THE FINITE-DIFFERENCE METHOD**

$t_j$	$x(t_j) - x_{j,1} = e_{j,1}$	$x(t_j) - x_{j,2} = e_{j,2}$	$x(t_j) - x_{j,3} = e_{j,3}$	$x(t_j) - x_{j,4} = e_{j,4}$
	$h_1 = 0.2$	$h_2 = 0.1$	$h_3 = 0.05$	$h_4 = 0.025$
0.0	0.000000	0.000000	0.000000	0.000000
0.2	0.002847	0.000704	0.000176	0.000044
0.4	0.005898	0.001460	0.000364	0.000091
0.6	0.009007	0.002229	0.000556	0.000139
0.8	0.012013	0.002974	0.000742	0.000185
1.0	0.014780	0.003660	0.000913	0.000228
1.2	0.017208	0.004263	0.001063	0.000265
1.4	0.019235	0.004766	0.001189	0.000297
1.6	0.020815	0.005160	0.001287	0.000322
1.8	0.021920	0.005435	0.001356	0.000338
2.0	0.022533	0.005588	0.001394	0.000348

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2.2	0.022639	0.005615	0.001401	0.000350
2.4	0.022232	0.005516	0.001377	0.000344
2.6	0.021304	0.005285	0.001319	0.000329
2.8	0.019852	0.004926	0.001230	0.000308
3.0	0.017872	0.004435	0.001107	0.000277
3.2	0.015362	0.003812	0.000951	0.000237
3.4	0.012322	0.003059	0.000763	0.000191
3.6	0.008749	0.002171	0.000541	0.000135
3.8	0.004641	0.001152	0.000287	0.000072
4.0	0.000000	0.000000	0.000000	0.000000

**TABLE IV**

**EXTRAPOLATION OF THE NUMERICAL APPLICATIONS  $\{X_{j,1}\}, \{X_{j,2}\}, \{X_{j,3}\}$  OBTAINED WITH THE FINITE DIFFERENCE METHOD.**

$t_j$	$\frac{4x_{j,2} - x_{j,1}}{3} = z_{j,1}$	$\frac{4x_{j,3} - x_{j,2}}{3} = z_{j,2}$	$\frac{16z_{j,2} - z_{j,1}}{15}$	$x(t_j)$ Exact solution
0.0	1.250000	1.250000	1.250000	1.250000
0.2	1.317360	1.317351	1.317350	1.317350
0.4	1.326524	1.326506	1.326504	1.326505
0.6	1.281792	1.281764	1.281762	1.281762
0.8	1.189451	1.189414	1.189412	1.189412
1.0	1.056932	1.056889	1.056886	1.056886
1.2	0.892138	0.892090	0.892086	0.892086
1.4	0.703003	0.702951	0.702947	0.702948
1.6	0.497246	0.497191	0.497187	0.497187
1.8	0.282244	0.282188	0.282184	0.282184
2.0	0.064991	0.064935	0.064931	0.064931
2.2	-0.147918	-0.147973	-0.147977	-0.147977
2.4	-0.350268	-0.350322	-0.350325	-0.350325
2.6	-0.536207	-0.536258	-0.536261	-0.536261
2.8	-0.700213	-0.700259	-0.700263	-0.700262
3.0	-0.837072	-0.837113	-0.837116	-0.837116
3.2	-0.941850	-0.941885	-0.941888	-0.941888
3.4	-1.009870	-1.009898	-1.009899	-1.009899
3.6	-1.036688	-1.036707	-1.036708	-1.036708
3.8	-1.018075	-1.018085	-1.018086	-1.018086
4.0	-0.950000	-0.950000	-0.950000	-0.950000
0.0	1.250000	1.250000	1.250000	1.250000



# International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 2, Issue 10, October 2013

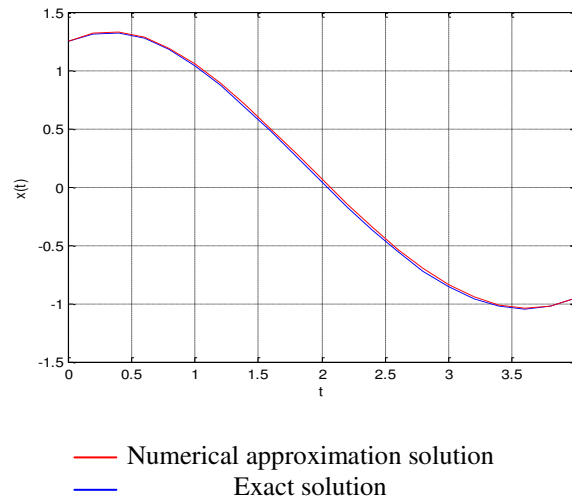


Fig. I Comparing the solution of numerical approximation solution, exact solution and solution of the finite difference method using MATLAB program to the given boundary value problem

### III. CONCLUSION

A boundary value problem is solved using finite difference method and is verified with exact solution. It is found that the results are agreed with exact solution.

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